Interest Rate Swap Valuation Since the Financial Crisis: Theory and Practice

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ABSTRACT

The financial crisis of 2007-09 revealed the importance of counterparty credit risk in the valuation of non-collateralized interest rate swaps. In theory, these valuations rest on assumed default probabilities and recovery rates. These assumptions, however, should be reflected in the risk-adjusted discount rates of the counterparties. Thus, in practice, swap valuations can be generated by discounting prospective swap settlements using risk-adjusted discount rates, cash flow by cash flow. This article demonstrates this method, discerning risk-adjusted discount rates from data that are readily available on the Bloomberg information system. Critically, if the inputs for the two methodologies are mutually consistent, theory and practice should yield identical valuations.

Introduction

The financial crisis of 2007-09 marked a fundamental change in methodologies used for valuing LIBOR-based interest rate swaps. Prior to the crisis, both dealers and end-users used a *single curve* methodology to determine the fair value for these derivatives. The exercise required projecting future cash flows based on a LIBOR forward curve and then discounting these projected cash flows using discount rates from that same LIBOR-based yield curve. With the bankruptcy of Lehman brothers, however, it became widely appreciated that this single curve methodology failed to address the issue of counterparty credit risk appropriately for these contracts. Moreover, accounting rules specifically initiated the requirement that derivatives be carried on the balance sheet at fair values that reflect their credit quality; but this requirement was generally not satisfied by the single curve methodology.

Since then, practitioners have largely coalesced around two different methods to value interest rate swaps. Both methods start with a LIBOR forward curve, but after that common starting step, the two methods take different approaches with respect to addressing credit risk. The more intuitive approach uses the standard discounted cash flow (DCF) analysis, applying discount rates that reflect the risk premiums applicable to the owing party, cash flow by cash flow. Thus, this "risk-adjusted DCF" approach uses up to three yield curves: the LIBOR yield curve (to generate the expected cash flows) and the yield curves pertaining to each of the parties to the contract (to derive discount factors for each of the parties with an obligation to pay).

The alternative methodology employs a two-step process for valuing interest rate swaps. We call this approach the *ex-post adjustment* (EPA) method. Under this approach, we first value the derivative without considering credit quality. That is, we discount projected cash flows using discount factors derived from overnight indexed swaps (OIS) curve[2] to generate a starting swap valuation that assumes a probability of default that approaches zero. These discount rates reflect spot and forward overnight fed funds rates, and they are typically taken to be proxies for risk-free interest rates. We then modify this initial valuation by making further adjustments that relate to credit considerations.

When applying the EPA method to a generalized swap where cash flows move in both directions (i.e., some inflows and some outflows), a credit valuation adjustment (CVA) would be made relating to cash flows expected to be received and a distinct debit valuation adjustment (DVA) would be made for cash flows

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[2] OIS discounting has become standard practice to get a value for a swap assuming no default. See Hull and White (2013) and Smith (2013).

expected to be paid. Thus, the CVA for one party would be a DVA for the other, and vice versa. Given the reciprocal nature of CVAs and DVAs, it's not uncommon for many market participants to ignore the distinction and simply refer to both adjustments as CVA adjustments. In any case, as both adjustments serve to shrink the values of their associated risk-free cash flows, whether valuing the swap from the perspective of the asset position or the liability position, the same *absolute value* would result for both parties. A positive value would reflect an asset position, while a negative value would reflect a liability.

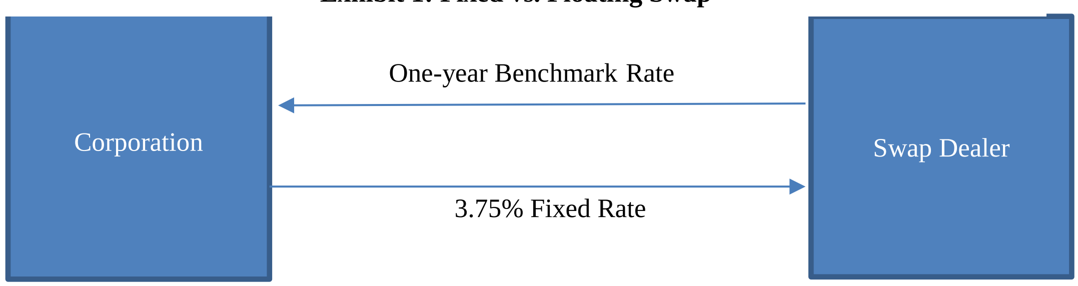
It should be clear that if we start with a valuation generated by the risk-adjusted DCF method, we could solve for respective default probabilities and recovery assumptions such that the DCF valuation could be replicated using the EPA method. Alternatively, starting with an EPA valuation, for any given CVA and DVA, we can solve for a unique credit spread for the owing party, such that if we had a consistent data set, the two valuations, again, would be identical.[3]

The first section of this paper presents the current state of valuation theory where we pose a numerical example to demonstrate the way forward interest rates, discount factors, and swap market values may be determined. Then, in the second section, we introduce practical considerations to demonstrate the way in which analysts may employ readily accessible data to generate their desired valuations.

Interest Rate Swap Valuation in Theory

Our numerical example assumes a five-year swap with a fixed rate of 3.75%, where resets and settlements based on an assumed one-year accrual period and benchmark rate. We assume the counterparties are a corporate end-user (fixed payer) and a swap dealer (fixed receiver). The annual accrual period is simply a pedagogic expedient to simplify our math by eliminating the need for showing fractional time periods in our equations. The swap is pictured in Exhibit 1 in the familiar box-and-arrow format.

Exhibit 1: Fixed vs. Floating Swap



Settlements in this swap are determined on a net basis in arrears. That means that if the one-year benchmark rate were 2.00% on any reset date (i.e., at the start of the accrual period), the corporation would pay the dealer 1.75% of the swap's notional principal at the end of the year: (3.75% - 2.00%) = 1.75%. If the one-year rate were 5.00%, the cash flow would move in the opposite direction. That is, the dealer would pay the corporation 1.25% of the notional in arrears: (5.00% - 3.75%) = 1.25%.

A key step in this swap valuation exercise is to project the forward rates for the floating reference rate. These forward rates would be observable if there were an actively traded futures market for one-year benchmark rates. Otherwise, forward rates would likely be inferred from observed data on one-year through five-year annual payment benchmark bonds. The procedure to get these rates is called *bootstrapping*. Exhibit 2 displays the assumed coupon rates and prices on these benchmark bonds.

[3] In recent years, some money-center banks have begun to include other risks and costs to their derivatives business in addition to the CVA and DVA. The term "XVA" has emerged to summarize these valuation adjustments. These include the FVA (funding valuation adjustment), KVA (capital valuation adjustment), LVA (liquidity valuation adjustment), and MVA (initial margin valuation adjustment). See Gregory (2015), Green (2016), Ruiz (2015), Lu (2016), and Smith (2017) for detailed discussion of these adjustments, as well as a few more.

Exhibit 2: Market Data on Benchmark Bonds (Par = 100)

Projected Settlements	Coupon Rate	Price
1	0.00%	99.75
2	0.25%	99.25
3	1.50%	100.125
4	1.75%	98.25
5	2.75%	100.25

We first derive benchmark discount factors that are consistent with the principle of no arbitrage. These discount factors, denoted DF_1 through DF_5, are obtained iteratively by applying the principle that the observed bond price of a zero-coupon bond equals its future value times its associated present value factor. Thus, we solve for the one-year discount factor, DF_1, as follows:

$$99.75 = 100 * DF_1; \qquad DF_1 = 0.997500$$

The hallmark of a bootstrapping procedure is that the output from one step is used as an input in the next step. That is, in the next period (reflecting the two-year bond), we discount the coupon paid at the end of year 1 using DF_1, and then at the second year, the second (and final) coupon payment plus par is discounted by the second discount factor, DF_2. [Note: all calculations are done on a spreadsheet and rounded results are reported in the text].

$$99.25 = (0.25 * 0.997500) + (0.25 + 100) * DF_2; \qquad DF_2 = 0.987537$$

Continuing iteratively for each bond, we determine the remaining discount factors as follows:

$$100.125 = (1.50 * 0.997500) + (1.50 * 0.987537) + (101.50 * DF_3); \quad DF_3 = 0.957118$$

$$98.25 = (1.75 * 0.997500) + (1.75 * 0.987537) + (1.75 * 0.957118) + (101.75 * DF_4); \qquad DF_4 = 0.915000$$

$$100.25 = (2.75 * 0.997500) + (2.75 * 0.987537) + (2.75 * 0.957118) + (2.75 * 0.915000) + (102.75 * DF_5); \qquad DF_5 = 0.872436$$

Given discount factors for each date, we are now able to derive associated one-year forward rates, denoted as $R_{1,2}$, $R_{2,3}$, $R_{3,4}$, and $R_{4,5}$. For example, $R_{4,5}$ is the one-year rate between dates 4 and 5 that is consistent with no arbitrage opportunities in the benchmark bond market. The spot one-year rate on the yield curve is simply the yield to maturity on the one-year zero-coupon bond, determined as follows:

$$P_1*(1+R_{0,1}) = 100 \text{ implies } R_{0,1} = (100/P_1) - 1 \text{ or } R_{0,1} = (100/99.75) - 1 = 0.2506\%$$

where P_1 is the spot price for the one-year bond, and $R_{0,1}$ is its yield to maturity. The one-year forward rates that follow that spot rate are calculated from the following progression of equations:

$$DF_2*(1+R_{0,1})*(1+R_{1,2})=1$$

$$DF_3*(1+R_{0,1})*(1+R_{1,2})*(1+R_{2,3})=1$$

$$DF_4*(1+R_{0,1})*(1+R_{1,2})*(1+R_{2,3})*(1+R_{3,4})=1$$

$$DF_5*(1+R_{0,1})*(1+R_{1,2})*(1+R_{2,3})*(1+R_{3,4})*(1+R_{4,5})=1$$

Recall, however, that $(1+R_{0,1}) = 1/DF_1$; and thus:

$R_{1,2} = DF_1/DF_2 - 1 = 0.997500/0.987537 - 1 = 0.010088$ $R_{1,2} = 1.0088\%$

$R_{2,3} = DF_2/DF_3 - 1 = 0.987537/0.957118 - 1 = 0.031783$ $R_{2,3} = 3.1783\%$

$R_{3,4} = DF_3/DF_4 - 1 = 0.957118/0.915000 - 1 = 0.046030$ $R_{3,4} = 4.6030\%$

$R_{4,5} = DF_4/DF_5 - 1 = 0.915000/0.872436 - 1 = 0.048787$ $R_{4,5} = 4.8787\%$

Exhibit 3 displays the projected cash flows for the swap assuming a notional principal of 100. Values shown in columns 2 through 4 reflect amounts from the perspective of the corporate entity – i.e., the fixed-rate payer. Positive values reflect cash inflows to the corporate; negative values reflect cash outflows. The projected variable payments reflect the one-year spot and subsequent forward rates derived above, times the notional principal (= 100). The contractual fixed payments from the corporation to the dealer reflect the 3.75% fixed rate, also times the notional principal. Finally, the fourth column shows the projected *net* settlements (i.e., the sum columns 2 and 3).

Exhibit 3: Cash Flows for the Swap (Notional Principal = 100)

Year	Variable Settlements	Contractual Fixed Settlements	Projected Net Settlements
1	0.2506	-3.7500	-3.4994
2	1.0088	-3.7500	-2.7412
3	3.1783	-3.7500	-0.5717
4	4.6030	-3.7500	0.8530
5	4.8787	-3.7500	1.1287

Because the corporation and the swap dealer are not default-free entities, we need to discount the projected net settlement payments using *risk-adjusted discount factors*. These discount factors are found by performing another bootstrapping procedure that incorporates assumptions about the credit risk parameters for the two counterparties. The corporation is assumed to have a 1.50% probability of default for each year (known as the *hazard rate*) – *conditional* on not defaulting in a prior period. If a default were to occur, however, some portion of the exposure would likely be recovered. We assume a recovery rate of 40% for the corporate entity. The swap dealer, on the other hand, might reasonably be expected to have a lower annual probability of default and a lower recovery rate. We arbitrarily assume values of 0.50% and 10%, respectively, for these inputs. In practice, the credit risk parameters can be inferred from the prices of credit default swaps or received directly from a credit rating agency or consultancy.[4]

For illustrative purposes, we solve for the risk-adjusted price of a five-year zero-coupon corporate bond in Exhibit 4. While we describe this process for only one maturity, this same procedure needs to be repeated to solve for the risk-adjusted prices for zero-coupon bonds with maturities associated with all the settlements throughout the horizon of the swap under consideration – for both the corporate entity and the swap dealer.

[4] Hull (2014) provides an example of valuing a credit default swap (CDS) given assumptions about the conditional probability of default and the recovery rate. Given the observed price for the CDS, the example could be revised to back out the probability of default consistent with the assumption for recovery. For an example of a consultancy that provides probabilities of default, see Kamakura Corporation at kamakuraco.com. See Jarrow (2012) for caveats about simple use of credit default swaps to get the default probabilities.

Exhibit 4: CVA Computation for a Five-Year Zero-Coupon Corporate Bond

Year	Spot and Forward Rates	Exposure	LGD	POD	DF	CVA
1	0.2506%	87.4623	52.4774	1.5000%	0.997500	0.7852
2	1.0088%	88.3446	53.0068	1.4775%	0.987537	0.7734
3	3.1783%	91.1525	54.6915	1.4553%	0.957118	0.7618
4	4.6030%	95.3482	57.2089	1.4335%	0.915000	0.7504
5	4.8787%	100.0000	60.0000	1.4120%	0.872436	0.7391
				7.2783%		**3.8099**

To calculate the CVA adjustment, we use the previously derived benchmark spot and forward rates to determine the default loss exposure for each year. We start with the most distant settlement date in column 3 and work up the table, year by year. If the corporation were to default at the end of the fifth year, the investor could lose the entire par value (100). However, if the default were to occur at the end of the fourth year, the exposure would be 95.3482, which is 100 discounted back for one year using the benchmark forward rate of 4.8787% ($R_{4,5}$). That is, 95.3482 = 100/1.048787. For default at the end of year 3, the exposure is 91.1525 = 100/(1.048787 * 1.046030), etc.

Given the year-by-year exposures, our loss given default (LGD) value simply reflects the expected loss that will ultimately be realized with a default. In our example, we assume a recovery rate of 40%, such that the LGD values in column 4 are the exposures for default loss for each year, times (1 – 0.40). This amount, (1 – 0.40), is known as the *loss severity*.

Next, we calculate probabilities of default for year n (POD_n) in column 5, as follows:

$$POD_n = \left(1 - \sum_{i=0}^{n-1} POD_i\right) * 0.0150\%$$

This equation is used, iteratively, year by year. Again, by assumption, the first year's POD is 1.50% such that the probability of survival is 98.50%. In general, the survival probability for each year is the parenthetical term in the above equation. The POD for the second year, conditional on no prior default, is 1.50% * (100 - 1.50)% = 1.50% * 98.50% = 1.4775%. Analogous calculations of the above equation are required to calculate the PODs for all remaining accrual periods. Given these respective PODs, the *cumulative probability of default* over the lifetime of the bond is their sum – in this example, 7.2783%. Finally, we reproduce the previously calculated benchmark discount factors in column 6 of Exhibit 4, enabling the calculation of the annual contributions to CVA by multiplying the LGD times the POD times the DF in each year. These products are shown in the far-right column of Exhibit 4. The CVA for the five-year zero-coupon corporate bond is the sum of this column, 3.8099.

This same methodology is required to calculate cumulative CVAs for the complete set of zero-coupon corporate bonds having maturities ranging from the first accrual period through the last, which, in this case, means one year to five years. These respective CVAs are shown in Exhibit 5. We subtract the corporate CVAs from the corresponding benchmark zero-coupon bond prices, which are just the benchmark discount factors multiplied by 100. From the zero-coupon corporate bond prices we obtain the discount factors that apply to the fixed-rate payer on the interest rate swap. For example, the five-year benchmark zero-coupon bond price is 87.2436. That less the CVA of 3.8099 gives a five-year zero-coupon bond price of 83.4337 (= 87.2436 – 3.8099) and a discount factor of 0.834337**.** Analogous content is shown in Exhibit 6 relating to the calculations of the discount factors relating to the credit risk of the swap dealer, the fixed-rate receiver.[5]

[5] Note that the back-up calculations shown in the text pertain exclusively to the determination of the corporate discount factor for the discrete five-year horizon. Analogous processing (not reported) is required for all settlement date horizons.

Exhibit 5: CVA Computation for the Corporate Entity (Five years; Par = 100)

Year	Benchmark DF	Benchmark Bond Price	Corporate CVA	Corporate Bond Price	Corporate DF
1	0.997500	99.7500	0.8978	98.8522	0.9885
2	0.987537	98.7537	1.7642	96.9895	0.9699
3	0.957118	95.7118	2.5456	93.1662	0.9317
4	0.915000	91.5000	3.2206	88.2794	0.8828
5	0.872436	87.2436	3.8099	83.4337	0.8343

Exhibit 6: CVA Computation for the Dealer (Five years; Par = 100)

Year	Benchmark DF	Benchmark Bond Price	Dealer CVA	Dealer Bond Price	Dealer DF
1	0.997500	99.7500	0.4489	99.3011	0.9930
2	0.987537	98.7537	0.8866	97.8671	0.9787
3	0.957118	95.7118	1.2857	94.4261	0.9443
4	0.915000	91.5000	1.6347	89.8653	0.8987
5	0.872436	87.2436	1.9434	85.3002	0.8530

Given these discount factors, the most direct method to incorporate credit considerations into the valuation of an interest rate swap is to identify the counterparty projected to make the net settlement payment and then use the discount factor corresponding to that date and entity. We show this in Exhibit 7, which displays the projected net settlements from the perspective of the corporate fixed-payer (Exhibit 3), the discount factors that apply to the corporation (Exhibit 5), and the discount factors for the swap dealer (Exhibit 6). The first three net settlements are negative—they are liabilities of the corporation and are discounted using the corporate discount factors. The last two are positive—they are assets to the corporation and are discounted using the swap dealer's discount factors. The net result is a swap value of -4.9212 to the corporation and + 4.9212 to the swap dealer.

$$(-3.4994 * 0.988522) + (-2.7412 * 0.969895) + (-0.5717 * 0.931662)$$

$$+ (0.8530 * 0.898653) + (1.1287 * 0.853002) = -4.9212$$

Exhibit 7: Risk Adjusted Swap Valuation (Corporate Perspective)

Year	Projected Net Settlements	Corporate DF	Dealer DF	Swap PV
1	-3.4994	0.988522	0.993011	-3.4592
2	-2.7412	0.969895	0.978671	-2.6587
3	-0.5717	0.931662	0.944261	-0.5326
4	0.8530	0.882794	0.898653	0.7666
5	1.1287	0.844337	0.853002	0.9628
				-4.9212

Interest Rate Swap Valuation in Practice

To value a swap using the credit-adjusted discounted cash flow method, we simply apply the standard theoretical valuation model using discount factors that reflect the credit quality of the owing counterparty, cash flow by cash flow. Our discussion uses the Swap Manager function on Bloomberg to value an interest rate swap on 6/30/16. The swap in question has an end date of 12/31/16, a fixed coupon of 1.4875%, and a variable leg tied to one-month LIBOR. Settlements are netted monthly, reflecting the actual/360-day count convention. These features are captured in Exhibit 8 and Exhibit 9, reflecting the valuations first from the perspective of the *fixed payer* and then from the perspective of the *fixed receiver*. Note that the same underlying curve selections and conventions apply to both screens, such that both show NPVs of the same absolute value. The positive value reflects the swap being an asset for the fixed rate payer; the negative value reflects the swap being a liability for the fixed rate receiver.

The next three Bloomberg screen shots show the cash flow calculations from the perspective of the fixed-rate payer. Exhibit 10 shows the fixed cash flows; Exhibit 11 shows the variable cash flows; and Exhibit 12 puts them together to show the net cash flows, along with their discounted values.[6] The variable and fixed settlements shown in these exhibits are found by multiplying the notional amount, times the respective interest rates, times time. In our example, we assume the actual/360 day-count method for calculating time, but some swaps may apply alternative day-count conventions.[7] Present value amounts are simply the product of payment amounts times their associated discount factors.

[6] In each of these exhibits, when the last day of the month happens to be a weekend day, which occurs in July and December), the settlement date and accrual period end reverts by convention to the last business day of that month.

[7] Note that some confusion may arise in terms of terminology, as ISDA swap definitions identify the *reset date* as the first date in which the new variable rate will be applied, while Bloomberg calls the reset date the date at which the new variable rate is determined.

Exhibit 8: Main Screen - Fixed Rate Payer

Exhibit 9: Main Screen - Fixed Rate Receiver

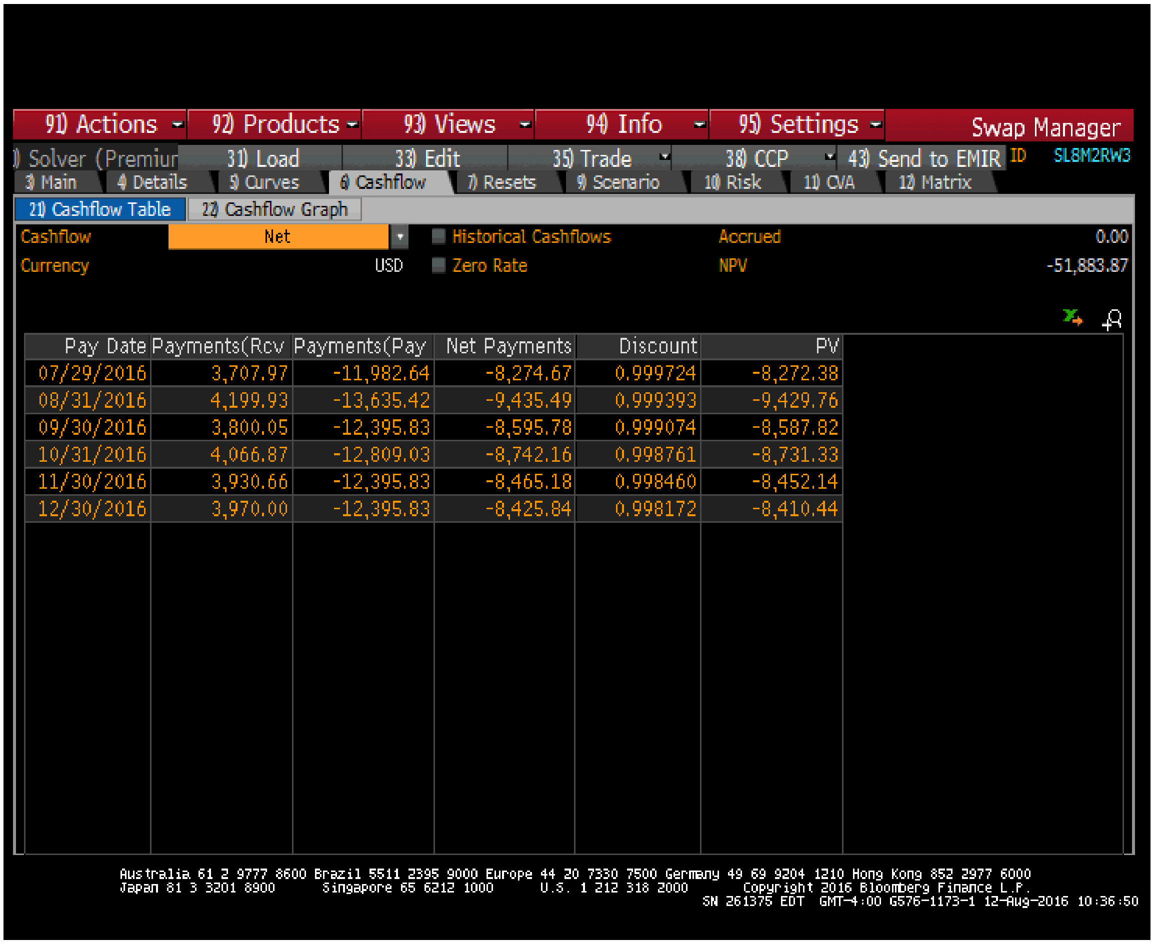
Exhibit 10: Fixed Cash Flows

Pay Date	Accrual Start	Accrual End	Days	Notional	Principal	Payment	Discount	PV
07/29/2016	06/30/2016	07/29/2016	29	-10,000,000.00	0.00	-11,982.64	0.999724	-11,979.33
08/31/2016	07/29/2016	08/31/2016	33	-10,000,000.00	0.00	-13,635.42	0.999393	-13,627.14
09/30/2016	08/31/2016	09/30/2016	30	-10,000,000.00	0.00	-12,395.83	0.999074	-12,384.35
10/31/2016	09/30/2016	10/31/2016	31	-10,000,000.00	0.00	-12,809.03	0.998761	-12,793.15
11/30/2016	10/31/2016	11/30/2016	30	-10,000,000.00	0.00	-12,395.83	0.998460	-12,376.75
12/30/2016	11/30/2016	12/30/2016	30	-10,000,000.00	0.00	-12,395.83	0.998172	-12,373.18

Exhibit 11: Variable Cash Flows

Pay Date	Accrual Start	Accrual End	Days	Notional	Principal	Reset Date	Reset Rate	Payment
07/29/2016	06/30/2016	07/29/2016	29	10,000,000.00	0.00	06/28/2016	0.46030	3,707.97
08/31/2016	07/29/2016	08/31/2016	33	10,000,000.00	0.00	07/27/2016	0.45817	4,199.93
09/30/2016	08/31/2016	09/30/2016	30	10,000,000.00	0.00	08/26/2016	0.45601	3,800.05
10/31/2016	09/30/2016	10/31/2016	31	10,000,000.00	0.00	09/28/2016	0.47228	4,066.87
11/30/2016	10/31/2016	11/30/2016	30	10,000,000.00	0.00	10/27/2016	0.47168	3,930.66
12/30/2016	11/30/2016	12/30/2016	30	10,000,000.00	0.00	11/28/2016	0.47640	3,970.00

Exhibit 12: Net Cash Flows



Pay Date	Payments(Rcv	Payments(Pay	Net Payments	Discount	PV
07/29/2016	3,707.97	-11,982.64	-8,274.67	0.999724	-8,272.38
08/31/2016	4,199.93	-13,635.42	-9,435.49	0.999393	-9,429.76
09/30/2016	3,800.05	-12,395.83	-8,595.78	0.999074	-8,587.82
10/31/2016	4,066.87	-12,809.03	-8,742.16	0.998761	-8,731.33
11/30/2016	3,930.66	-12,395.83	-8,465.18	0.998460	-8,452.14
12/30/2016	3,970.00	-12,395.83	-8,425.84	0.998172	-8,410.44

With this foundation, we now must perform the following steps:

1. Infer the risk-free OIS interest rates that underlie the discount factors.
2. Identify the yield curve of any owning party of the swap. (Only one yield curve would apply if all the cash flow obligations are in one direction. Two yield curves would be required if both parties expect to have a net payment obligation.)
3. Determine the spread between the risk-free rate and the rate pertaining to the owing party (i.e., the risk premium) at the maturity corresponding to the time remaining until the swap's end date.
4. To find the risk-adjusted discount rate for each cash flow, add this risk premium of the owing party to that cash flow's risk-free discount rate.
5. Given the set of risk-adjusted discount rates, calculate the discount factors for each of the component net settlements.
6. Find the risk-adjusted PV by multiplying each anticipated net settlement by its respective risk-adjusted discount factor, and sum the elements.

We discuss each step, in turn.

Given that the yield curves that we will shortly be referencing reflect interest rates that employ the semi-annual compounding convention, we derive the underlying discount rate from the following formula:

$$DF = \frac{1}{\left(1 + \frac{Rate}{2}\right)^{\left(days/\left(\frac{365}{2}\right)\right)}} \tag{1}$$

Rate = The implied risk-free discount rate, annual yield, semiannual compounding
DF = The risk-free discount factor
days = The total number of days between the valuation date and the settlement date.

In equation (1), *Rate*/2 is the rate per semiannual period, a time frame that has 182.5 (= 365/2) days.[8] The denominator is the future value per unit invested for the fraction of the semiannual period. Therefore, the exponent to (1 + *Rate*/2) is days/182.5.

Equation (2) rearranges (1) to isolate *Rate*:

$$Rate = \left(\left(\frac{1}{DF}\right)^{\left(\left(\frac{365}{2}\right)/days\right)} - 1\right) * 2 \tag{2}$$

In the general case where we cannot expect to find actively traded credit derivatives that would provide risk premiums for the required maturities, we're forced to rely on aggregated data to extract reasonable estimates for the required risk premia. Various data providers publish such information. For instance, Bloomberg provides curves for assorted credit ratings in the following sectors:

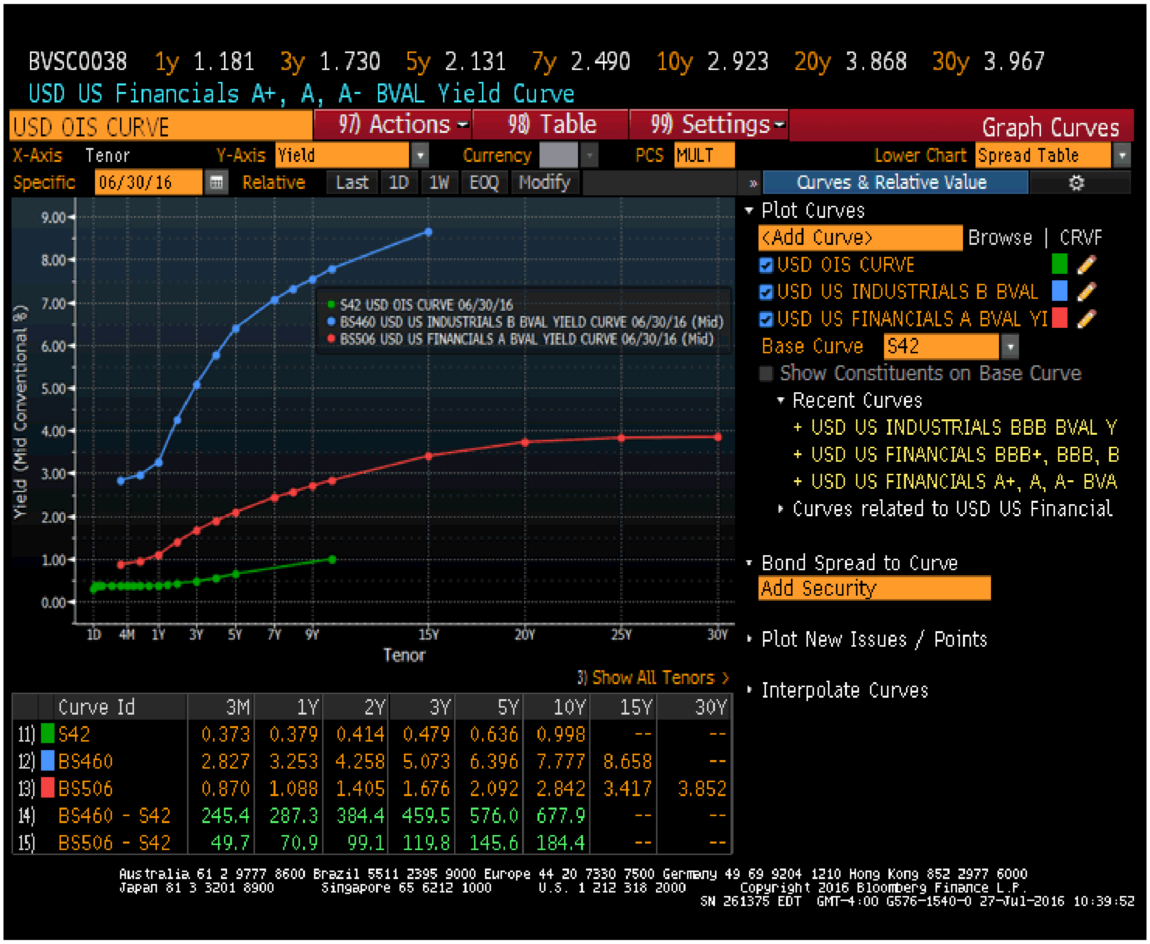
- Communications
- Consumer Discretionary
- Consumer Stables
- Energy
- Financials
- Government and Agency
- Health Care
- Industrials
- Materials
- Technology
- Utilities

Bloomberg yield curves are constructed with rates that reflect closing prices each day for senior unsecured fixed-rate bond financing in each of the respective sectors. For the purposes of our example, the parties to the trade are assumed to be an A-rated financial firm (e.g., the dealer) and a B-rated industrial (e.g., an end-

[8] The assumption of a common accrual period length for all periods (i.e., 182.5 days) may not be universally adopted. That is, some analysts might choose to measure the length of each accrual period, precisely.

user). Exhibit 13 reproduces the screen shot from Bloomberg, reflecting these curves as of the desired value date (6/30/16).

Exhibit 13: Sector Yield Curves



In our example, the time remaining until the swap's end date is six months. Thus, we interpolate results for the three-month horizon and the one-year horizon. For the A-rated financial firm, the three-month yield is shown in the bottom line of Exhibit 13 to be 49.7 basis points and the one-year yield is 70.9 basis points. The interpolated six-month yield is thus 56.8 basis points, calculated using simple weights of 2/3 and 1/3. That is, (2/3 * 49.7) + (1/3 * 70.9) = 56.8. For the B-rated industrial, the three-month and one-year yields are 245.4 and 287.3 basis points, respectively, in the line above giving an interpolated six-month rate of 259.4 basis points, (2/3 * 245.4) + (1/3 * 287.3) = 259.4.

As the swap's value will differ depending on which party bears the obligation to pay the fixed rate, we generate two sets of adjusted discount rates – one associated with the situation where the A-rated financial firm has the obligation to pay, and the other where the B-rated industrial has that obligation. Given each of the respective risk-adjusted discount rates, we find their associated risk-adjusted discount factors using equation (3), which is a restatement of equation (1). The credit-risk-adjusted discount factor (*AdjDF*) corresponds to the risk-adjusted rate (*AdjRate*), which is the sum of the implied OIS rate for each settlement date and the interpolated credit risk premium.

$$AdjDF = \frac{1}{\left(1 + \frac{AdjRate}{2}\right)^{\left(days/\left(\frac{365}{2}\right)\right)}} \qquad (3)$$

Results of applying this equation are shown for both counterparties and all settlement dates in Exhibit 14.

Exhibit 14: Risk Adjusted Discount Factors

Valuation Date:	6/30/16						
Accrual End & Settlement	**Days to Settlement**	**OIS Discount Factor**	**Implied OIS Rate**	**Spread for Financial A**	**Financial A Adjusted Discount Factor**	**Spread for Industrial B**	**Industrial B Adjusted Discount Factor**
7/29/16	29	0.9997	0.3477%	0.568%	0.9995	2.594%	0.9980
8/31/16	62	0.9994	0.3578%	0.568%	0.9990	2.594%	0.9956
9/30/16	92	0.9991	0.3679%	0.568%	0.9986	2.594%	0.9935
10/31/16	123	0.9988	0.3682%	0.568%	0.9981	2.594%	0.9914
11/30/16	153	0.9985	0.3680%	0.568%	0.9976	2.594%	0.9893
12/30/16	183	0.9982	0.3653%	0.568%	0.9972	2.594%	0.9872

For example, the 92-day OIS discount factor associated with the third settlement is 0.9991 (from Bloomberg Exhibits 10 and 12). That value corresponds to a rate of 0.3679%, using equation (2):

$$Rate = \left(\left(\frac{1}{0.9991}\right)^{\left(\left(\frac{365}{2}\right)/92\right)} - 1\right) * 2 = 0.003679$$

The interpolated yield for the A-rated financial firm is 0.568% (56.8 basis points), as calculated above. Using equation (3) gives an adjusted discount rate of 0.9986:

$$AdjDF = \frac{1}{\left(1 + \frac{0.00568}{2}\right)^{\left(92/\left(\frac{365}{2}\right)\right)}} = 0.9986$$

The remaining implied OIS rates and the adjusted discount factors for Financial A and Industrial B in Exhibit are calculated in the same manner.

Given these adjusted discount rates, we can now value the swap under two alternative assumptions in Exhibit 15: (1) that the A-rated financial firm bears the obligation to pay, and (2) that the B-rated industrial bears the burden to pay.

Under the prevailing market conditions of this example, the fixed-rate *payer* (irrespective of credit quality) would be carrying the swap as a liability, while the fixed-rate *receiver* would be in the asset position. Thus, if the fixed-rate payer is the A-rated financial entity, the swap's absolute value will be $51,784.33 for both the asset and the liability; and if the fixed payer is the B-rated industrial, the common asset/liability value would be $51,466.99.

Exhibit 15: Risk Adjusted Valuations

Fixed Rate:	1.4875%	(paid by A-rated Financial firm)					
Valuation Date:	6/30/16	Notional:	10,000,000				
Accrual End & Settlement	**Accrual Days**	**Projected 1-month LIBOR Reset Rate**	**Variable Settlement**	**Fixed Settlement**	**Net Settlement**	**Financial A Adjusted Discount**	**PV**
7/29/16	29	0.460%	3,708	(11,983)	(8,275)	0.9995	(8,268)
8/31/16	62	0.458%	4,200	(13,635)	(9,436)	0.9990	(9,419)
9/30/16	92	0.456%	3,800	(12,396)	(8,596)	0.9986	(8,574)
10/31/16	123	0.472%	4,067	(12,809)	(8,742)	0.9981	(8,712)
11/30/16	153	0.472%	3,931	(12,396)	(8,465)	0.9976	(8,429)
12/30/16	183	0.476%	3,970	(12,396)	(8,426)	0.9972	(8,383)
							(51,784)

Fixed Rate:	1.4875%	(paid by B-rated Industrial firm)					
Valuation Date:	6/30/16	Notional:	10,000,000				
Accrual End & Settlement	**Accrual Days**	**Projected 1-month LIBOR Reset Rate**	**Variable Settlement**	**Fixed Settlement**	**Net Settlement**	**Industrial B Adjusted Discount**	**PV**
7/29/16	29	0.460%	3,708	(11,983)	(8,275)	0.9980	(8,254)
8/31/16	62	0.458%	4,200	(13,635)	(9,436)	0.9956	(9,385)
9/30/16	92	0.456%	3,800	(12,396)	(8,596)	0.9935	(8,528)
10/31/16	123	0.472%	4,067	(12,809)	(8,742)	0.9914	(8,650)
11/30/16	153	0.472%	3,931	(12,396)	(8,465)	0.9893	(8,355)
12/30/16	183	0.476%	3,970	(12,396)	(8,426)	0.9872	(8,295)
							(51,467)

It bears repeating that while this example reflects consistency of the paying party throughout the remaining life of the swap, in the general case, some net cash flows might be in one direction and others in the opposite direction. In such a case, the applicable adjusted discount rates would reflect the factor associated with the paying party, cash flow by cash flow.

In determining the appropriate risk premia, the methodology described herein relies on yield curves that relate to unsecured senior debt. Moreover, the objective data we use are an aggregation of portfolios of bond that exhibit at least some degree of heterogeneity. Individual users may perceive their circumstances to be better or worse than those reflected in the available yield curves, and in those cases, some subjective adjustments to the inputs may reasonably be justified.

Conclusions

Since the financial crisis, market participants have been required to address counterparty credit risk explicitly when valuing interest rate swaps for presentation in financial reports. We discuss two methods to accomplish this: the ex-post adjustment (EPA) method and the risk-adjusted discounted cash flow (DCF) method. With the EPA method, the projected settlement payments are initially discounted using OIS rates, which are proxies for "risk-free" rates; and then a further adjustment for credit risk is made to this starting valuation. The DCF method, on the other hand, uses the same projected payments under the swap, but the risk-adjusted valuation is found using discount rates that reflect the credit quality of the owing counterparty, settlement-by-settlement.

The challenge for both methods is determining the relevant inputs. The EPA calculation requires estimates for the probabilities of default and recovery rates for the parties of the swap, and the DCF method requires a determination of appropriate risk-adjusted discount rates. If the underlying data sets for these two methods happen to be mutually consistent, these two methodologies would yield identical results, but such consistency is most unlikely.

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